Primordial Magnetic Helicity

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  - Intensities: $B \sim \mu\mathrm{Gauss}$.
  - Coherence scale: the size of the structure
- Intergalactic voids are probably endowed with rather intense magnetic fields.

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  - Inflationary Mechanisms: low intensities but large coherence lengths.

Field Topology

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- Magnetic Helicity (Chern-Simon number) accounts for the field topology: twists and links of field lines.

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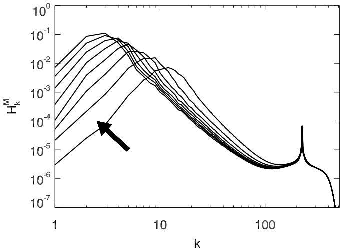
- Conservation of H^M everely constraints dynamo action
- Helical fields decay slower than non-helical ones.

Magnetic Helicity

- Peforms an inverse cascade: H^M stored in small scales evolves toward large scales



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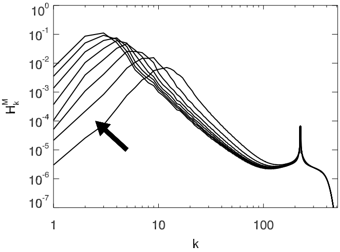

- Magnetic Energy and Magnetic Helicity spectra dimensionally related as $E_k^M \sim kH_k^M$: Magnetic energy is dragged toward large scales, enabling the field to reorganize coherently.

H^M Generation During Reheating

Model

- charged scalar field minimally coupled to gravity

$$\mathcal{L} = \sqrt{-g}\left[g^{\mu\nu} D_\mu \Phi D_\nu^\dagger \Phi^\dagger + m^2 \Phi\Phi^\dagger + \frac{1}{4} F^{\mu\nu} F_{\mu\nu}\right] \qquad (3)$$

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$$a_I(t) \propto \exp(Ht) \to a_R(t) \propto t^{2/3} \quad (4)$$

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- scalar field vacuum state turns into particle state (particle creation):

$$\phi_I\left(\kappa,\eta\right) = \alpha_\kappa\phi_R\left(\kappa,\eta\right) + \beta_\kappa\phi_I^*\left(\kappa,\eta\right) \quad (5)$$

β_κ accounts for particle creation.

- Sub-horizon modes:

$$\beta_p \sim \frac{i}{16\tau_0 p^5}, \qquad p \geq 1 \tag{6}$$

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- Super-horizon modes:

$$\beta_p \simeq -i\sqrt{\frac{H}{2m}}\sqrt{\frac{9}{8}}p^{-3/2}, \qquad p < m/H \tag{7}$$

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- Contribution of subhorizon modes is suppressed relative to the superhorizon ones. But it is responsible for a mildly turbulent flow on scales of the order of the horizon size, with Reynolds numbers $R_e \simeq 100$.

- stochastic currents J^i appear:

$$
\begin{aligned}
\langle J^i(\bar{r}, t) \rangle &= 0 \\
\langle J^i(\bar{r}, t)\, J^i(\bar{r}', t') \rangle &\neq 0
\end{aligned}
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- Magnetic helicity of random fields:

$$\Xi(\mathcal{V},\eta,\eta') \equiv \langle \mathcal{H}_M(\mathcal{V},\eta), \mathcal{H}_M(\mathcal{V},\eta')\rangle \tag{9}$$

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-

$$\begin{aligned} \Xi\left(\mathcal{V},\eta,\eta'\right) &= \int d^3\kappa' \int d^3\kappa \int_{\mathcal{V}} d^3x' \int_{\mathcal{V}} d^3x \\ &\quad \exp\left(i\bar{\kappa}\cdot\bar{x}\right)\exp\left(i\bar{\kappa}'\cdot\bar{x}'\right)\Xi\left(\bar{\kappa},\bar{\kappa}',\eta,\eta'\right) \end{aligned}$$

- $$\Xi\left(\bar{\kappa},\bar{\kappa}',\eta,\eta'\right)=\left\langle\mathcal{H}_M\left(\bar{\kappa},\eta\right)\mathcal{H}_M\left(\bar{\kappa}',\eta'\right)\right\rangle \tag{10}$$

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- $$\mathcal{H}_M\left(\bar{\kappa},\eta\right)=\ \mathcal{A}_i\left(\bar{p},\eta\right)\mathcal{B}^i\left(\bar{\kappa}-\bar{p},\eta\right) \tag{11}$$

- $$\mathcal{A}_i\left(\bar{p},\eta\right) = G_{ret}\left(\eta-\tau_1,\bar{p}\right)\delta\left(\bar{p}-\bar{p}_1\right)\mathcal{J}_i\left(\bar{p}_1,\tau_1\right) \qquad (12)$$

$$\begin{aligned}\mathcal{B}_i\left(\bar{\kappa}-\bar{p},\eta\right) &= i\epsilon^{ils}G_{ret}\left(\eta-\tau_2,\bar{\kappa}-\bar{p}\right)\delta\left(\bar{\kappa}-\bar{p}-\bar{p}_2\right) \\ &\quad \left(\kappa_l-p_l\right)\mathcal{J}_s\left(\bar{p}_2,\tau_2\right) \qquad (13)\end{aligned}$$

- $$\mathcal{A}_i(\bar{p},\eta) = G_{ret}(\eta - \tau_1, \bar{p})\,\delta(\bar{p} - \bar{p}_1)\,\mathcal{J}_i(\bar{p}_1, \tau_1) \tag{12}$$

  $$\begin{aligned}\mathcal{B}_i(\bar{\kappa} - \bar{p}, \eta) \quad &= \quad i\epsilon^{ils} G_{ret}(\eta - \tau_2, \bar{\kappa} - \bar{p})\,\delta(\bar{\kappa} - \bar{p} - \bar{p}_2) \\ &\quad (\kappa_l - p_l)\,\mathcal{J}_s(\bar{p}_2, \tau_2) \end{aligned} \tag{13}$$

- $$\begin{aligned}\mathcal{J}_i(\bar{p},\tau) \quad &= \quad ie\delta(\bar{q}_1 + \bar{q}_2 - \bar{p})\,\delta(\tau - \varsigma_1)\,\delta(\tau - \varsigma_2) \\ &\quad (q_{1i} - q_{2i})\,\phi^1(\bar{q}_1, \varsigma_1)\,\phi^2(\bar{q}_2, \varsigma_2) \end{aligned} \tag{14}$$

- $$\begin{aligned}\mathcal{H}_M\left(\bar{\kappa},\eta\right) &= -ie^2 G_{ret}\left(\eta-\tau_1,\bar{p}\right) G_{ret}\left(\eta-\tau_2,\bar{\kappa}-\bar{p}\right) \\ &\quad \delta\left(\tau_1-\varsigma_1\right)\delta\left(\tau_1-\varsigma_2\right)\delta\left(\tau_2-\varsigma_3\right)\delta\left(\tau_2-\varsigma_4\right) \\ &\quad \delta\left(\bar{p}-\bar{p}_1\right)\delta\left(\bar{\kappa}-\bar{p}-\bar{p}_2\right)\delta\left(\bar{q}_1+\bar{q}_2-\bar{p}_1\right) \\ &\quad \delta\left(\bar{q}_3+\bar{q}_4-\bar{p}_2\right) \qquad (15) \\ &\quad \epsilon^{ils}\left(q_{1i}-q_{2i}\right)\left(q_{3s}-q_{4s}\right)\left(\kappa_l-p_l\right) \\ &\quad \phi^1\left(\bar{q}_1,\varsigma_1\right)\phi^2\left(\bar{q}_2,\varsigma_2\right)\phi^1\left(\bar{q}_3,\varsigma_3\right)\phi^2\left(\bar{q}_4,\varsigma_4\right)\end{aligned}$$

After integrating out the time delta functions the magnetic helicity correlation function spectrum reads

$$\begin{aligned}
\Xi\left(\bar{\kappa},\bar{\kappa}',\eta,\eta'\right) \quad = \quad & -e^4 G_{ret}\left(\eta-\tau_1,\bar{p}\right) G_{ret}\left(\eta-\tau_2,\bar{\kappa}-\bar{p}\right) \\
& G_{ret}\left(\eta'-\tau_1',\bar{p}'\right) G_{ret}\left(\eta'-\tau_2',\bar{\kappa}'-\bar{p}'\right) \\
& \delta\left(\bar{p}-\bar{p}_1\right)\delta\left(\bar{\kappa}-\bar{p}-\bar{p}_2\right)\delta\left(\bar{p}'-\bar{p}_1'\right) \\
& \delta\left(\bar{\kappa}'-\bar{p}'-p_2'\right)\delta\left(\bar{q}_1+\bar{q}_2-\bar{p}_1\right)\delta\left(\bar{q}_3+\bar{q}_4-\bar{p}_2\right) \\
& \delta\left(\bar{q}_1'+\bar{q}_2'-\bar{p}_1'\right)\delta\left(\bar{q}_3'+\bar{q}_4'-\bar{p}_2'\right) \qquad (16) \\
& \epsilon^{abc}\left(q_{1a}-q_{2a}\right)\left(q_{3c}-q_{4c}\right)\left(\kappa_b-p_b\right) \\
& \epsilon^{def}\left(q_{1d}'-q_{2d}'\right)\left(q_{3f}'-q_{4f}'\right)\left(\kappa_e'-p_e'\right) \\
& \left\langle\phi^1\left(\bar{q}_1,\tau_1\right)\phi^1\left(\bar{q}_3,\tau_2\right)\phi^1\left(\bar{q}_1',\tau_1'\right)\phi^1\left(\bar{q}_3',\tau_2'\right)\right\rangle \\
& \left\langle\phi^2\left(\bar{q}_2,\tau_1\right)\phi^2\left(\bar{q}_4,\tau_2\right)\phi^2\left(\bar{q}_2',\tau_1'\right)\phi^2\left(\bar{q}_4',\tau_2'\right)\right\rangle
\end{aligned}$$

- Each scalar field can be decomposed as:

$$\phi^i(\bar{q},\tau) = \phi^+(\bar{q},\tau) + \phi^-(\bar{q},\tau) = \phi(\bar{q},\tau)\, a_q + \phi^*(\bar{q},\tau)\, a^\dagger_{-q} \tag{17}$$

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- The only terms that contribute to the mean value are

$$\begin{aligned}
&\left\langle \phi^1(\bar{q}_1,\tau_1)\,\phi^1(\bar{q}_3,\tau_2)\,\phi^1(\bar{q}'_1,\tau'_1)\,\phi^1(\bar{q}'_3,\tau'_2)\right\rangle \to \\
&\left\langle \phi^+(\bar{q}_1,\tau_1)\,\phi^+(\bar{q}_3,\tau_2)\,\phi^-(\bar{q}'_1,\tau'_1)\,\phi^-(\bar{q}'_3,\tau'_2)\right\rangle \\
&+\left\langle \phi^+(\bar{q}_1,\tau_1)\,\phi^-(\bar{q}_3,\tau_2)\,\phi^+(\bar{q}'_1,\tau'_1)\,\phi^-(\bar{q}'_3,\tau'_2)\right\rangle
\end{aligned} \tag{18}$$

- The first term on the r.h.s. of expression (18) gives

$$\left\langle \phi^+ \left(\bar{q}_1, \tau_1\right) \phi^+ \left(\bar{q}_3, \tau_2\right) \phi^- \left(\bar{q}_1', \tau_1'\right) \phi^- \left(\bar{q}_3', \tau_2'\right) \right\rangle = \frac{1}{2} G^+ \left(\bar{q}_1, \bar{q}_1', \tau_1, \tau_1'\right) G^+ \left(\bar{q}_3, \bar{q}_3', \tau_2, \tau_2'\right) + \frac{1}{2} G^+ \left(\bar{q}_1, \bar{q}_3', \tau_1, \tau_2'\right) G^+ \left(\bar{q}_3, \bar{q}_1', \tau_2, \tau_1'\right)$$

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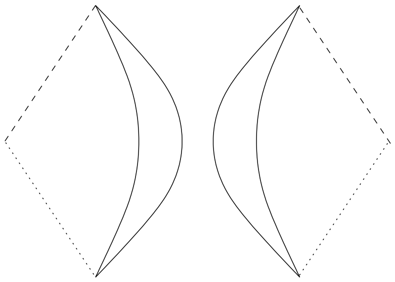
$$\left\langle \phi^{+}\left(\bar{q}_1,\tau_1\right)\phi^{+}\left(\bar{q}_3,\tau_2\right)\phi^{-}\left(\bar{q}'_1,\tau'_1\right)\phi^{-}\left(\bar{q}'_3,\tau'_2\right)\right\rangle =$$
$$\frac{1}{2}G^{+}\left(\bar{q}_1,\bar{q}'_1,\tau_1,\tau'_1\right)G^{+}\left(\bar{q}_3,\bar{q}'_3,\tau_2,\tau'_2\right)$$
$$+\frac{1}{2}G^{+}\left(\bar{q}_1,\bar{q}'_3,\tau_1,\tau'_2\right)G^{+}\left(\bar{q}_3,\bar{q}'_1,\tau_2,\tau'_1\right)$$

- while the second term gives

$$\left\langle \phi^{+}\left(\bar{q}_1,\tau_1\right)\phi^{-}\left(\bar{q}_3,\tau_2\right)\phi^{+}\left(\bar{q}'_1,\tau'_1\right)\phi^{-}\left(\bar{q}'_3,\tau'_2\right)\right\rangle =$$
$$G^{+}\left(\bar{q}_1,\bar{q}_3,\tau_1,\tau_2\right)G^{+}\left(\bar{q}'_1,\bar{q}'_3,\tau'_1,\tau'_2\right)$$

- "Mean Helicity" graph: Multiplicity 1



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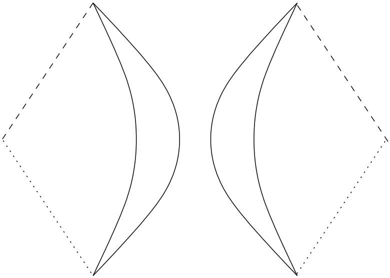
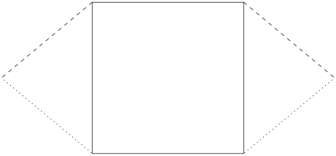


- the prefactor is identically null.

- "Square" graph: Multiplicity 4



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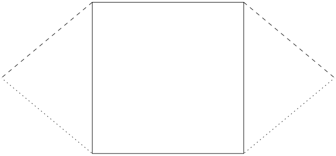
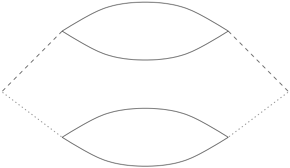


- $$\xi_s\left(\bar{\kappa},\eta,\eta'\right) = -2e^4 G_{ret}\left(\eta-\tau_1,\bar{p}\right) G_{ret}\left(\eta-\tau_2,\bar{\kappa}-\bar{p}\right)$$
$$G_{ret}\left(\eta'-\tau_1',\bar{q}_1+\bar{q}_2'-\bar{\kappa}\right) G_{ret}\left(\eta'-\tau_2',-\bar{q}_1-\bar{q}_2'\right)$$
$$\left[\bar{q}_1\cdot\left(\bar{\kappa}\times\bar{p}\right)\right]\left[\bar{\kappa}\cdot\left(\bar{q}_1\times\bar{q}_2'\right)\right]$$
$$\phi\left(\bar{q}_1,\tau_1\right)\phi^*\left(-\bar{q}_1,\tau_2'\right)\phi\left(\bar{\kappa}-\bar{q}_1,\tau_2\right)\phi^*\left(\bar{q}_1-\bar{\kappa},\tau_1'\right)$$
$$\phi\left(\bar{p}-\bar{q}_1,\tau_1\right)\phi^*\left(\bar{q}_1-\bar{p},\tau_2\right)\phi\left(\bar{q}_2',\tau_1'\right)\phi^*\left(-\bar{q}_2,\tau_2'\right),$$

- "Two Bubbles" graph: Multiplicity 2



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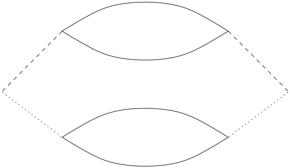
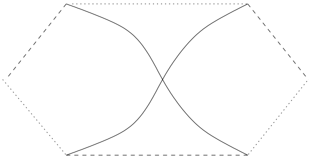


- $$\xi_{2b}\left(\kappa,\eta,\eta'\right) = e^4 G_{ret}\left(\eta-\tau_1,p\right) G_{ret}\left(\eta-\tau_2,\kappa-p\right)$$
$$G_{ret}\left(\eta'-\tau_1',-p\right) G_{ret}\left(\eta'-\tau_2',p-\kappa\right)$$
$$\left[(2\bar{q}_1-\bar{p})\cdot(\kappa\times\bar{q}_3)-2\bar{q}_1\cdot(\bar{p}\times\bar{q}_3)\right]^2$$
$$\phi\left(\bar{q}_1,\tau_1\right)\phi^*\left(-\bar{q}_1,\tau_1'\right)\phi\left(\bar{q}_3,\tau_2\right)\phi^*\left(-\bar{q}_3,\tau_2'\right)$$
$$\phi\left(\bar{p}-\bar{q}_1,\tau_1\right)\phi^*\left(\bar{q}_1-\bar{p},\tau_1'\right)\phi\left(\bar{\kappa}-\bar{p}-\bar{q}_3,\tau_2\right)$$
$$\phi^*\left(p+\bar{q}_3-\bar{\kappa},\tau_2'\right)$$

- "Cross" graph: Multiplicity 2



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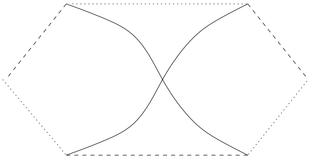

- $$\xi_c\left(\kappa,\eta,\eta'\right) = -e^4 G_{ret}\left(\eta-\tau_1,\bar{p}\right) G_{ret}\left(\eta-\tau_2,\bar{\kappa}-\bar{p}\right)$$
$$G_{ret}\left(\eta'-\tau'_1,\bar{q}_1-\bar{q}_3-\bar{p}\right) G_{ret}\left(\eta'-\tau'_2,-\bar{\kappa}+\bar{p}-\bar{q}_1+\bar{q}_3\right)$$
$$\left[\left(2\bar{q}_1-\bar{p}\right)\cdot\left(\bar{\kappa}\times\bar{q}_3\right)-2\bar{q}_1\cdot\left(\bar{p}\times\bar{q}_3\right)\right]$$
$$\left[\left(\bar{p}-\bar{q}_3\right)\cdot\left(\bar{\kappa}\times\bar{q}_1\right)-2\bar{q}_1\cdot\left(\bar{p}\times\bar{q}_3\right)\right]$$
$$\phi\left(\bar{q}_1,\tau_1\right)\phi^*\left(-\bar{q}_1,\tau'_2\right)\phi\left(\bar{q}_3,\tau_2\right)\phi^*\left(-\bar{q}_3,\tau'_1\right)$$
$$\phi\left(\bar{p}-\bar{q}_1,\tau_1\right)\phi^*\left(\bar{q}_1-\bar{p},\tau'_1\right)\phi\left(\bar{\kappa}-\bar{p}-\bar{q}_3,\tau_2\right)$$
$$\phi^*\left(-\bar{\kappa}+\bar{p}+\bar{q}_3,\tau'_2\right)$$

Some Dimensions

$$
\begin{aligned}
\frac{m}{H} &\simeq 10^{-11} - 10^{-9} \\
\kappa_G &\simeq 10^{-51} - 10^{49} \\
\frac{\sigma_0}{H} &\simeq 10^{-7/2} - 10^{-3/2}
\end{aligned}
$$

H^M *Generation During Reheating*

H^M *on Large Scales* $(\kappa \ll p, q)$ *due to Smooth and Fluctuating Fields* $(p, q \lesssim m/H)$

- $$\Xi\left(\kappa\right)) \sim \int_{V(\kappa)} \int_{V(\kappa)} \int_{\kappa_0}^{\kappa} \xi\left(\kappa'\right) d^3\kappa' \tag{19}$$

  κ_0^{-1}: scale of the largest homogenous patch created during Inflation.

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- $$\xi\left(\kappa\right) \sim 8\left(\frac{H}{\sigma_0}\right)^4 \left(\frac{m}{H}\right)^{11/3} \frac{6^4}{\left(\pi^4 e\right)^{4/3}} \kappa^2 \qquad (20)$$

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- $$\mathcal{H}_M(\kappa) \equiv \Xi^{1/2}(\kappa) \sim \left(\frac{H}{\sigma_0}\right)^2 \left(\frac{m}{H}\right)^{11/6} \frac{102}{\left(\pi^4 e\right)^{2/3}} \kappa^{-1/2} \tag{22}$$

  Fractal dimension $D = 1/2$.

H^M Generation During Reheating

H^M on Large Scales due to Only Smooth Fields ($\kappa \sim p, q \sim \kappa_G$) : Seed Field for Galactic Dynamo

- $$\mathcal{H}_M(\kappa_\lambda) \sim e^{-2/3}\left(\frac{m}{H}\right)^{4/3}\left(\frac{H}{\sigma_0}\right)^2 \qquad (23)$$

  Fractal dimension $D = 0$

H^M *Generation During Reheating*

H^M on Large Scales due to Only Smooth Fields ($\kappa \sim p, q \sim \kappa_G$) : Seed Field for Galactic Dynamo

- $$\mathcal{H}_M(\kappa_\lambda) \sim e^{-2/3} \left(\frac{m}{H}\right)^{4/3} \left(\frac{H}{\sigma_0}\right)^2 \tag{23}$$

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- This expression coincides with the direct estimation from the magnetic field correlation function $\langle B_i(\kappa) B^i(\kappa)\rangle$
  For a galactic scale today: $B_G^{hel} \sim 10^{-61}$ Gauss.

H^M Generation During Reheating

Magnetic Helicity at Small Scales: causal evolution of the field inside particle horizon

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- Magnetic helicity on a scale $\kappa^{-1} \sim 1$ at the beginning of radiation dominance:

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$$\mathcal{B}_{eq} \sim \frac{1}{\pi^{2/3} e^{5/3}} \left(\frac{H}{m}\right)^{1/2} \left(\frac{h_{eq}}{h_{rh}}\right)^{-1/6} \sim 10^{-9}\mathrm{Gauss}\text{–}10^{-5}\mathrm{Gauss} \tag{26}$$

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- Regular fields on cosmological scales are fully helical, but of very small intensities.
- Fields on small scales ($\sim$ horizon at reheating) may grow to interesting intensities and scales, provided that turbulence is operative during radiation dominance.