The primordial bispectrum and the collapse of the wave function

Gabriel León

Departamento de Física - Universidad de Buenos Aires

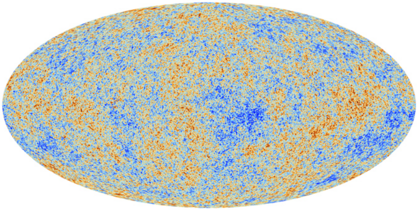
April 2014

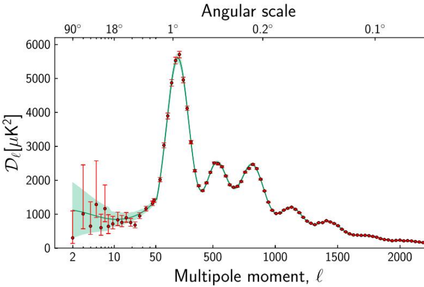
Joint work with Dra. Susana Landau (Univ. de Buenos Aires), Dr. Daniel Sudarsky (ICN-UNAM)

Outline

Successes of the Inflationary Paradigm

- Solves the problems of the Hot Big Bang model (Horizon, Flatness, etc.) [Guth, 1981]
- The main success...
  Quantum Fluctuations of the inflaton results in theoretical predictions of the **spectrum of primordial inhomogeneities**. [Mukhanov, Chibisov 1982]
- Observational Data in agreement with such predictions!





The Data

- The temperature anisotropies of the CMB photons emitted by the Last Scattering Surface are related to the **curvature perturbation** Ψ (i.e., the **scalar metric perturbation**, in the appropriate gauge) by

$$\frac{\delta T}{T_0}(\theta, \varphi) \simeq \frac{1}{3}\Psi(\eta_D, \mathbf{x}_D) \tag{1}$$

- Moreover, one can use the spherical harmonic functions and write $\frac{\delta T}{T_0}(\theta, \varphi) = \sum_{m,l} a_{lm} Y_{lm}(\theta, \varphi)$. Thus,

$$a_{lm} = \frac{1}{3}\int d\Omega \Psi(\eta_D, \mathbf{x}_D) Y^{\star}_{lm}(\theta, \varphi) \tag{2}$$

  This is the quantity that is measured.
- The inflationary paradigm is supposed to provide a physical mechanism to relate Ψ with the quantum fluctuations of the matter fields in the early Universe.

Theoretical Predictions

- Quantum fluctuations lead to a quantum description of the curvature perturbation $\hat{\Psi}$.
- One proceeds to calculate the 2-point correlation function in the vacuum state $|0\rangle$:

$$\langle 0|\hat{\Psi}(\eta,\vec{x})\hat{\Psi}(\eta,\vec{y})|0\rangle \tag{3}$$

- Once $\langle 0|\hat{\Psi}(\eta,\vec{x})\hat{\Psi}(\eta,\vec{y})|0\rangle$ is known, one proceeds to make the identification

$$\langle 0|\hat{\Psi}(\eta,\vec{x})\hat{\Psi}(\eta,\vec{y})|0\rangle = \overline{\Psi(\eta,\vec{x})\Psi(\eta,\vec{y})} \tag{4}$$

- For slow roll inflation ($\epsilon \equiv M_P^2/2(\partial_\phi V/V)^2$)

$$\mathcal{P}_\Psi(k,\eta) = \frac{1}{2\pi^2}|\Psi_k(\eta)|^2 k^3 \propto \left(\frac{H^2}{M_p^2\epsilon}\right)\Bigg|_{k=aH} \tag{5}$$

Theoretical Predictions

- Quantum fluctuations lead to a quantum description of the curvature perturbation $\hat{\Psi}$.
- One proceeds to calculate the 2-point correlation function in the vacuum state $|0\rangle$:

$$\langle 0|\hat{\Psi}(\eta,\vec{x})\hat{\Psi}(\eta,\vec{y})|0\rangle \tag{3}$$

- Once $\langle 0|\hat{\Psi}(\eta,\vec{x})\hat{\Psi}(\eta,\vec{y})|0\rangle$ is known, one proceeds to make the identification

$$\langle 0|\hat{\Psi}(\eta,\vec{x})\hat{\Psi}(\eta,\vec{y})|0\rangle = \overline{\Psi(\eta,\vec{x})\Psi(\eta,\vec{y})} \tag{4}$$

- For slow roll inflation ($\epsilon \equiv M_P^2/2(\partial_\phi V/V)^2$)

$$\mathcal{P}_\Psi(k,\eta) = \frac{1}{2\pi^2}|\Psi_k(\eta)|^2 k^3 \propto \left(\frac{H^2}{M_P^2\epsilon}\right)\Big|_{k=aH} \tag{5}$$

Theoretical Predictions

- Quantum fluctuations lead to a quantum description of the curvature perturbation $\hat{\Psi}$.
- One proceeds to calculate the 2-point correlation function in the vacuum state $|0\rangle$:

$$\langle 0|\hat{\Psi}(\eta,\vec{x})\hat{\Psi}(\eta,\vec{y})|0\rangle \tag{3}$$

- Once $\langle 0|\hat{\Psi}(\eta,\vec{x})\hat{\Psi}(\eta,\vec{y})|0\rangle$ is known, one proceeds to make the identification

$$\langle 0|\hat{\Psi}(\eta,\vec{x})\hat{\Psi}(\eta,\vec{y})|0\rangle = \overline{\Psi(\eta,\vec{x})\Psi(\eta,\vec{y})} \tag{4}$$

- For slow roll inflation ($\epsilon \equiv M_P^2/2(\partial_\phi V/V)^2$)

$$\mathcal{P}_\Psi(k,\eta) = \frac{1}{2\pi^2}|\Psi_k(\eta)|^2 k^3 \propto \left(\frac{H^2}{M_p^2\epsilon}\right)\Bigg|_{k=aH} \tag{5}$$

Traditional non-Gaussianity

- In the traditional approach, the NG of the curvature perturbation is characterized by the 3-point correlation function $\langle\hat{\Psi}(x)\hat{\Psi}(y)\hat{\Psi}(z)\rangle = \overline{\Psi(x)\Psi(y)\Psi(z)}$
- NG in the CMB is characterized by the **angle-averaged bispectrum**

$$B_{l_1 l_2 l_3} = \sum_{m_i} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \overline{a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}}. \tag{6}$$

- Assuming the simplest inflationary model (i.e., single slow-roll scalar field in the Bunch-Davies vacuum state with canonical kinetic term), the expression for the bispectrum is

$$B_{l_1 l_2 l_3} = S(l_1, l_2, l_3) f_{\rm NL} A^2 \tag{7}$$

- $A \simeq H^2/M_P^2\epsilon$ and the parameter f_{NL}, which characterizes the amplitude of NG, are fixed by the observational data; $A \simeq 10^{-9}$ and $f_{\rm NL} = 2.7 \pm 5.8$.

Something is not clear I

- It is not clear: $\langle 0|\hat{\Psi}(\eta,\vec{x}_1)\ldots\hat{\Psi}(\eta,\vec{x}_n)|0\rangle = \overline{\Psi(\eta,\vec{x}_1)\ldots\Psi(\eta,\vec{x}_n)}$

  "These are quantum averages, not averages over an ensemble of classical field configurations...some sort of decoherence must set in...It is not apparent just how this happen..."

  [S. Weinberg, *Cosmology* (2008)]
- Moreover, the overline strictly denotes an average over an ensemble of "universes."
- How do we go from an **ensemble average** to an average in **our own universe**?
- What does it mean to consider "Gaussian or non-Gaussian" aspects in the statistics of the ensemble and why does this translates into the same statistical aspects in our own universe?

Something is not clear II

- **Remarkable**: The universe was originally described by a space-time which is homogeneous and isotropic, and there is a scalar field (the inflaton) which is in a vacuum state also homogeneous and isotropic (there are some irrelevant deviations of this left from an imperfect inflation at the order of e^{-80}), but the universe ended up with inhomogeneities that fit the experimental data.
- **Question** How do we end up in a situation which is not symmetric (the symmetry being the homogeneity and isotropy) given that there is nothing in the dynamics that breaks such symmetries?
- In other words...

Something is not clear II

- **Remarkable**: The universe was originally described by a space-time which is homogeneous and isotropic, and there is a scalar field (the inflaton) which is in a vacuum state also homogeneous and isotropic (there are some irrelevant deviations of this left from an imperfect inflation at the order of e^{-80}), but the universe ended up with inhomogeneities that fit the experimental data.
- **Question** How do we end up in a situation which is not symmetric (the symmetry being the homogeneity and isotropy) given that there is nothing in the dynamics that breaks such symmetries?
- In other words...

How did it happen?

$$|\text{symmetric}\rangle \underbrace{\longrightarrow}_{i\hbar\partial_t|\psi\rangle = \hat{H}|\psi\rangle} |\text{non-symmetric}\rangle \qquad (\star)$$

> *"Quantum mechanical unitary evolution does not destroy translational invariance ... decoherence is not sufficient to explain the breaking of translational invariance. ...we have to appeal to either to Bohr's reduction postulate or to Everett's many-worlds interpretation of quantum mechanics. The first possibility does not look convincing in the cosmological context."*

[V. Mukhanov, Physical Foundations of Cosmology (2005)]

"Quantum mechanical unitary evolution does not destroy translational invariance … decoherence is not sufficient to explain the breaking of translational invariance. … we have to appeal to either to Bohr's reduction postulate or to Everett's many-worlds interpretation of quantum mechanics. The first possibility does not look convincing in the cosmological context."

[V. Mukhanov, Physical Foundations of Cosmology (2005)]

- The question (★) is often presented in the literature as: "How do quantum fluctuations become classical?"

"Quantum mechanical unitary evolution does not destroy translational invariance ... decoherence is not sufficient to explain the breaking of translational invariance. ... we have to appeal to either to Bohr's reduction postulate or to Everett's many-worlds interpretation of quantum mechanics. The first possibility does not look convincing in the cosmological context."

[V. Mukhanov, Physical Foundations of Cosmology (2005)]

- The question ($\star$) is often presented in the literature as: "How do quantum fluctuations become classical?"
- There are several answers, e.g., decoherence, many worlds interpretation of QM, consistent histories appraoch, etc. However, they all lack the physical mechanism to answer question ($\star$) because they are based on the standard formulation of QM.
- In the context of the **de Broglie-Bohm theory** the question ($\star$) has been addressed [N. Pinto-Neto *et al* (2012)].

"Quantum mechanical unitary evolution does not destroy translational invariance . . . decoherence is not sufficient to explain the breaking of translational invariance. . . . we have to appeal to either to Bohr's reduction postulate or to Everett's many-worlds interpretation of quantum mechanics. The first possibility does not look convincing in the cosmological context."

[V. Mukhanov, Physical Foundations of Cosmology (2005)]

- The question ($\star$) is often presented in the literature as: "How do quantum fluctuations become classical?"
- There are several answers, e.g., decoherence, many worlds interpretation of QM, consistent histories appraoch, etc. However, they all lack the physical mechanism to answer question ($\star$) because they are based on the standard formulation of QM.
- In the context of the **de Broglie-Bohm theory** the question ($\star$) has been addressed [N. Pinto-Neto *et al* (2012)].
- Often these issues are resolved as "just philosophy," with no impact whatsoever on the theoretical predictions. We will see that such preconception is mistaken

The transition and the collapse proposal

Our approach on attempting to answer the question (★) is described as:

The transition and the collapse proposal

Our approach on attempting to answer the question (★) is described as:

- Invoking the collapse of the wave function, one could break the symmetry of the original state.

$$|\text{symmetric}\rangle \quad \longrightarrow \textbf{Collapse} \longrightarrow \quad |\text{non-symmetric}\rangle \tag{8}$$

- According to standard QM the collapse of the wave function is the result of a **measurement**. But how do we apply this postulate in the cosmological setting?.

The transition and the collapse proposal

Our approach on attempting to answer the question (★) is described as:

- Invoking the collapse of the wave function, one could break the symmetry of the original state.

$$|\text{symmetric}\rangle \quad \longrightarrow \textbf{Collapse} \longrightarrow \quad |\text{non-symmetric}\rangle \tag{8}$$

- According to standard QM the collapse of the wave function is the result of a **measurement**. But how do we apply this postulate in the cosmological setting?.
- *Phenomenological Model*: **Some element intrinsic to the system** (possibly Quantum Gravity) induced the collapse of the wave function.
- In fact, concrete theories involving such ideas, designed to overcome the conceptual difficulties of quantum theory, have been proposed through the years (e.g. GRW, CSL, Diosi, Penrose, etc.)
- Even if we do not know precisely the nature of the collapse mechanism, we can parameterize the collapse (e.g. by using expectation values in the post-collapse state) and obtain testable predictions.

Detailed analysis of the collapse proposal

- Quantum field: $\hat{y} = a\hat{\delta\phi}$ (The metric perturbation Ψ is classical).
- Canonical conjugate momentum $\hat{\pi}_y = a\hat{\delta\phi}' = \hat{y}' - \hat{y}a'/a$.
- Einstein semiclassical equations:

$$\underbrace{G_{ab}}_{\text{classical}} = 8\pi G\langle \underbrace{\hat{T}_{ab}}_{\text{quantum}} \rangle \qquad \Rightarrow \qquad \Psi_k(\eta) = \frac{-4\pi G\phi_0'}{ak^2}\langle\hat{\pi}_k(\eta)\rangle$$

- **Before** the collapse $|0\rangle$:

$$\langle\hat{y}_k\rangle_0 = 0; \qquad \langle\hat{\pi}_k\rangle_0 = 0 \qquad \Rightarrow \qquad \Psi_k(\eta) = 0$$

- **After** the collapse $|0\rangle \to |\Theta\rangle$

$$\langle\hat{y}_k\rangle_\Theta \neq 0; \qquad \langle\hat{\pi}_k\rangle_\Theta \neq 0 \qquad \Rightarrow \qquad \Psi_k(\eta) \neq 0$$

The space-time (always classical) is no longer homogeneous and isotropic.

Parametrization of the collapse

- Explicitly

$$\langle \hat{\pi}_k^{R,I}(\eta_k^c) \rangle_\Theta = x_k^{R,I} \sqrt{(\Delta \hat{\pi}_k^{R,I})_0^2}, \tag{9}$$

  where R, I denotes the real and imaginary parts of the field respectively.
- η_k^c is the conformal time of collapse.
- $(\Delta \hat{\pi}_k^{R,I})_0^2$ are the uncertainties of each mode of the momentum.
- x_k^R, x_k^I are random variables with a Gaussian probability distribution function (PDF).

Theoretical Predictions

- Semiclassical gravity + collapse hypothsis

$$\Psi_{\mathbf{k}} = \left(\frac{1}{2k}\right)^{3/2} g(z_k) X_{\mathbf{k}}, \tag{10}$$

where $X_{\mathbf{k}} \equiv x^R_{\mathbf{k}} + i x^I_{\mathbf{k}}$ and $z_k \equiv k\eta^c_k$. If $X_{\mathbf{k}}$ is Gaussian, then $\Psi_{\mathbf{k}}$ is Gaussian.

- Our theoretical prediction for the a_{lm} is thus,

$$a^{\text{th}}_{lm} = \sum_{\mathbf{k}} \frac{F_{lm}(k)}{k^{3/2}} g(z_k) X_{\mathbf{k}}, \tag{11}$$

$F_{lm}(k)$ includes the transfer functions responsible for the acoustic oscillations.

- If you are interested in the details of the power spectrum and the observational data, in the collapse proposal, see **Maria Pia Piccirilli poster**.

The collapse bispectrum

- We will consider the quantity

$$\mathcal{B}_{l_1 l_2 l_3} \equiv \sum_{m_i} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} a_{1_1 m_1} a_{1_2 m_2} a_{1_3 m_3}. \quad (12)$$

- Note that it is different from the "traditional" bispectrum, there is no average over any sort of ensemble.
- Substituting our expression for a_{lm} (Eq. (11)) in terms of the random numbers, the collapse bispectrum yields

$$\mathcal{B}_{l_1 l_2 l_3} \equiv \sum_{m_i, \mathbf{k}_i} G_{l_1 m_1}(k_1) G_{l_2 m_2}(k_2) G_{l_3 m_3}(k_3) g(z_{k_1}) g(z_{k_2}) g(z_{k_3}) X_{\mathbf{k}_1} X_{\mathbf{k}_2} X_{\mathbf{k}_3}. \quad (13)$$

- Eq. (13) cannot be used to make a definite prediction as it involves the random numbers $X_{\mathbf{k}}$. However ...

The collapse bispectrum

- We can regard the sum of complex numbers appearing in Eq. (13) as representing a kind of two-dimensional random walk.
- Thus, we assume

$$|\mathcal{B}_{l_1 l_2 l_3}|^2_{\text{Our own universe}} \simeq |\mathcal{B}_{l_1 l_2 l_3}|^2_{\text{Most Likely}} \underbrace{\simeq}_{\text{Gaussian } X_{\mathbf{k}}} \overline{|\mathcal{B}_{l_1 l_2 l_3}|^2}$$

- **The collapse bispectrum** is of the form

$$|\mathcal{B}_{l_1 l_2 l_3}|_{\text{ML}} = A^{3/2} \Delta(l_1, l_2, l_3) \qquad (14)$$

  The data fixes $A \simeq 10^{-9}$, which comes from the collapse power spectrum.
- **The standard prediction** is given by

$$|B_{l_1,l_2,l_3}| = A^2 |f_{NL}| S(l_1, l_2, l_3).$$

  The observational data fixes $|f_{NL}| \simeq 10$

Conclusions

- We note that in obtaining the expression for $|\mathcal{B}_{l_1 l_2 l_3}|_{\text{M.L.}}$, we have assumed a Gaussian PDF for the random variable $X_{\mathbf{k}}$, this translates into a **Gaussian distribution for the metric perturbation** $\Psi_{\mathbf{k}}$.
- In other words, we have taken a **Gaussian curvature perturbation** and obtained a **non-vanishing prediction for the observed bispectrum**, while a Gaussian metric perturbation–*within the traditional inflationary paradigm*–would have yielded $|B_{l_1 l_2 l_3}| = 0$.
- Our model gives a precise shape and amplitude for the observed bispectrum, which can be verified or falsified by the data. No additional parameters are considered.
- From our point of view, the observed bispectrum corresponds to just one particular realization of a random quantum process (the self-induced collapse of the wave function). As we do not have access to other realizations (i.e. we do not have observational access to other universes) we cannot say anything conclusive as to whether or not the underlying PDF is Gaussian.