Regular phantom black holes as gravitational lenses

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Outline

- Uncharged regular and asymptotically flat phantom black holes are studied as gravitational lenses.
- The deflection angle in the strong deflection limit is obtained, from which the positions, magnifications and time delays of the relativistic images are calculated.
- Some of the results are briefly discussed and a comparison with the Schwarzschild and the vacuum Brans-Dicke cases is made.

Regular phantom black hole

- We consider the spherically symmetric geometry (Bronnikov and Fabris, 2006):

$$ds^2 = -A(r)dt^2 + B(r)dr^2 + C(r)d\Omega^2,$$

where

$$A(r) = B(r)^{-1} = 1 + \frac{3mr}{b^2} + (r^2 + b^2)\left[\frac{c}{b^2} + \frac{3m}{b^3}\tan^{-1}\left(\frac{r}{b}\right)\right],$$

$$C(r) = r^2 + b^2,$$

with c, m, and $b > 0$ constants.

- b is the scale of the scalar field.
- For $c = -3m\pi/2b$, the metric becomes asymptotically flat.
- The solution is regular everywhere.
- The constant m can be interpreted as the mass.

Regular phantom black hole

- Black hole solution for $m > 0$ (stable when $b = 3\pi m/2$), with one Killing horizon r_h. Region $r > r_h$ is asymptotically flat, and region $r < r_h$ is asymptotically de Sitter.
- Adopting the flatness condition, $m > 0$, and adimensionalizing with m,

$$x = r/m, \quad T = t/m, \quad \tilde{b} = b/m,$$

the metric takes the form

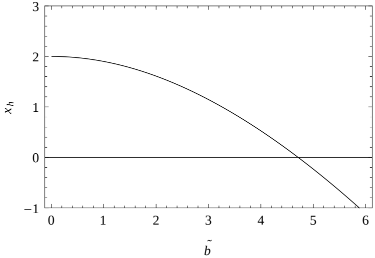
$$ds^2 = -A(x)dT^2 + B(x)dx^2 + C(x)(d\theta^2 + \sin^2\theta d\phi^2),$$

$$A(x) = B(x)^{-1} = 1 + \frac{3x}{\tilde{b}^2} + \frac{3}{\tilde{b}}\left(1 + \frac{x^2}{\tilde{b}^2}\right)\left[-\frac{\pi}{2} + \tan^{-1}\left(\frac{x}{\tilde{b}}\right)\right],$$

$$C(x) = x^2 + \tilde{b}^2.$$

Event horizon and photon sphere

- The adimensionalized radius of the horizon is a decreasing function of b/m



- The radius x_{ps} of the photon sphere is given by the largest positive solution of the equation

$$A'(x)C(x) = C'(x)A(x).$$

- For the regular phantom black hole, $x_{ps} = 3$.

Deflection angle

- The deflection angle for a photon coming from infinity, as a function of the closest approach distance x_0 is:

$$\alpha(x_0) = I(x_0) - \pi,$$

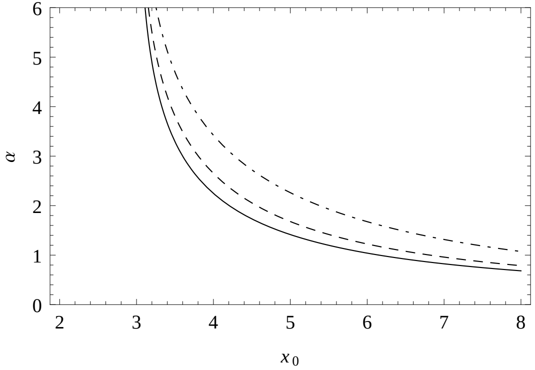
where

$$I(x_0) = \int_{x_0}^{\infty} \frac{2\sqrt{B(x)}dx}{\sqrt{C(x)}\sqrt{\frac{C(x)}{C(x_0)}\frac{A(x_0)}{A(x)} - 1}}.$$

- $\alpha(x_0)$ diverges when x_0 approaches to the radius of the photon sphere x_{ps}.

Deflection angle - massless black hole

- Plot of the exact deflection angle:



$\tilde{b} = 1$: solid line; $\tilde{b} = 3$: dashed line; $\tilde{b} = 6$: dashed-dotted line.

Approximations

The deflection angle can be analytically approximated by simple expressions in the following limits:

- Weak deflection limit: $x_0 \gg x_{ps}$. First non null order expansion in $1/x_0$, for primary and secondary images (Schneider et al., 1992).
- Strong deflection limit: For $0 < x_0/x_{ps} - 1 \ll 1$, we have (Bozza, 2002):

$$\alpha(x_0) \approx -a_1 \ln(x_0/x_{ps} - 1) + a_2,$$

with a_1 and a_2, constants.

  - $\alpha(x_0)$ diverges when $x_0 = x_{ps}$.
  - The two infinite sets of relativistic images can be obtained analytically.

The strong deflection limit

- It is useful to separate $I(x_0)$ as a sum of a divergent and a regular part:

$$I(x_0) = I_D(x_0) + I_R(x_0).$$

- I_D yields the leading order in the divergence of the deflection angle, which is logarithmic. I_R is a number that can be easily evaluated (Bozza, 2002).
- In the strong deflection limit and in terms of the impact parameter $u = \sqrt{C(x_0)/A(x_0)}$, the deflection angle results:

$$\alpha(u) = -c_1 \ln\left(\frac{u}{u_p s} - 1\right) + c_2 + O(u - u_{ps}),$$

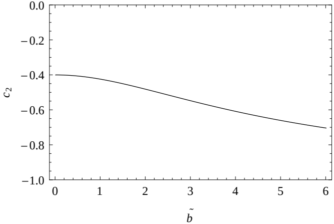
where u_{ps} is the impact parameter evaluated in $x_0 = x_{ps}$.

The strong deflection case: coefficients

- Performing the calculations for the regular phantom black hole, $c_1 = 1$ and c_2 is a decreasing function of $\tilde{b}$ (Eiroa and CMS, 2013):

$$c_2 = -\pi + c_R + \ln \frac{\tilde{b}^3 \left[-6\tilde{b} + 9\pi + \tilde{b}^2\pi - 2(9+\tilde{b}^2)\tan^{-1}\left(\frac{3}{\tilde{b}}\right) \right]^2}{(9+\tilde{b}^2)^2 \left[2\tilde{b} - 3\pi + 6\tan^{-1}\left(\frac{3}{\tilde{b}}\right) \right]^3},$$

where $c_R = I_R(x_{ps})$ is obtained numerically for each $\tilde{b}$:



Lens equation

- For the adimensionalized distances d_{os}, d_{ol} and d_{ls} (much greater than x_h), the lens equation has the form (Bozza, 2008):

$$\tan\beta = \frac{d_{ol}\sin\theta - d_{ls}\sin(\alpha-\theta)}{d_{os}\cos(\alpha-\theta)}.$$

- When the objects are highly aligned (small β), we have

$$\alpha = \pm 2n\pi \pm \Delta\alpha_n,$$

where $0 < \Delta\alpha_n \ll 1$, so the lens equation becomes

$$\beta = \theta \mp \frac{d_{ls}}{d_{os}}\Delta\alpha_n.$$

Positions of the relativistic images

- After replacing the strong deflection limit angle and performing a Taylor expansion around $\Delta\alpha_n = 0$, the angular positions of the relativistic images take the form:

$$\theta_n = \pm\theta_n^0 + \frac{\xi_n d_{os}}{d_{ls}}(\beta \mp \theta_n^0),$$

where

$$\theta_n^0 = \frac{u_{ps}}{d_{ol}}\left[1 + e^{(c_2 - 2n\pi))/c_1}\right],$$

and

$$\xi_n = \frac{u_{ps}}{c_1 d_{ol}} e^{(c_2 - 2n\pi)/c_1}.$$

Magnifications of the relativistic images

- Gravitational lensing conserves surface brightness (Schneider et al., 1992).
- For close alignment, the magnification of the n-th image is:

$$\mu_n = \left| \frac{\beta}{\theta_n} \frac{d\beta}{d\theta_n} \right|^{-1} .$$

- In the strong deflection limit, we obtain:

$$\mu_n = \frac{1}{\beta} \frac{\theta_n^0 \xi_n d_{os}}{d_{ls}} .$$

- The magnification decreases exponentially with n, so the first relativistic image is the brightest one.

Observables

- The outermost image is the first one (θ_1); the others, approach to the limiting value: $\theta_\infty = u_{ps}/d_{ol}$.
- The angular separation between the position of the first relativistic image and θ_∞ is:

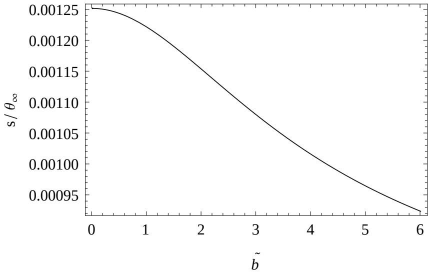
$$s = \theta_1 - \theta_\infty \overset{SDL}{=} \theta_\infty e^{(c_2 - 2\pi)/c_1}.$$

- The quotient between the flux of the first image (the brightest one) and the flux coming from all the others is given by:

$$r = \frac{\mu_1}{\sum_{n=2}^{\infty} \mu_n} \overset{SDL}{=} e^{2\pi/c_1}.$$

Observables

- For the regular phantom black hole, the observable r has the constant value $r = e^{2\pi}$.
- s/θ_∞ is plotted as a function of $\tilde{b}$:



Summary and discussion

- Uncharged regular phantom black holes, obtained from an action with a scalar field possessing a negative kinetic term and a potential, were studied as gravitational lenses.
- The strong deflection limit coefficients were calculated, which allowed to obtain analytical expressions for the positions and magnifications of the relativistic images, and the observables. Time delays can also be obtained.
- The first relativistic image is the outer one which is also the brightest of them, and the others are packed together at the limiting value θ_∞.
- If the strong deflection limit coefficients can be obtained from observational data, these phantom BHs studied can be clearly distinguished from the Schwarzschild and vacuum Brans-Dicke solutions, because the values of c_2 are different.

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